Sharing Online Advertising Revenue with Consumers

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Abstract. Online service providers generate much of their revenue by monetizing user attention through online advertising. In this paper, we investigate *revenue sharing*, where the user is rewarded with a portion of the surplus generated from the advertising transaction, in a cost-per-conversion advertising system. While revenue sharing can potentially lead to an increased user base, and correspondingly larger revenues in the long-term, we are interested in the effect of cashback in the short-term, in particular for a single auction. We capture the effect of cashback on the auction's outcome via *price-dependent conversion probabilities*, derived from a model of rational user behavior: this trades off the direct loss in per-conversion revenue against an increase in conversion rate. We analyze equilibrium behavior under two natural schemes for specifying cashback: as a fraction of the search engine's revenue per conversion, and as a fraction of the posted item price. This leads to some interesting conclusions: first, while there is an equivalence between the search engine and the advertiser providing the cashback specified as a fraction of search engine profit, this equivalence no longer holds when cashback is specified as a fraction of item price. Second, cashback can indeed lead to short-term increase in search engine revenue; however this depends strongly on the scheme used for implementing cashback *as a function* of the input. Specifically, given a particular set of input values (user parameters and advertiser posted prices), one scheme can lead to an increase in revenue for the search engine, while the others may not. Thus, an accurate model of the marketplace and the target user population is essential for implementing cashback.

1 Introduction

Advertising is the act of paying for consumers' attention: advertisers pay a publisher or service provider to display their ad to a consumer, who has already been engaged for another purpose, for example to read news, communicate, play games, or search. Consumers pay attention and receive a service, but are typically not directly involved in the advertising transaction. Revenue sharing, where the consumer receives some portion of the surplus generated from the advertising transaction, is a method of involving the user that could potentially lead to

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an increased user base for the service provider, albeit at the cost of a possible decrease in short-term revenue.

In May 2008, Microsoft introduced cashback in LiveSearch, where users who buy items using the LiveSearch engine receive cashback on their purchases. As in Livesearch, revenue sharing is best implemented in a *pay-per-conversion* system, where advertisers need to make a payment only when users actually purchase items– since money must change hands to trigger an advertising payment and a revenue share, the system is less susceptible to gaming by users as compared to systems based on cost-per-impression (CPM) or cost-per-click (CPC). Since revenue sharing in this setting corresponds directly to price discounts on items purchased, this gives users a direct incentive to engage with the advertisements on the page. Thus, there is in fact a potential for short-term revenue benefits to the search engine in the form of increased conversion probabilities, in addition to the possibility of an increased user base in the long term.

In this paper, we present a model to study the effect of revenue sharing on search engine revenues, and advertiser and user welfare in a single auction (specifically, we do not model long-term effects). We model the impact of cashback on the user via a price dependent conversion probability, and investigate equilibrium behavior in an auction framework. There are multiple natural schemes for revenue sharing: should cashback be specified as a fraction of the item price, or as a fraction of the search engine's profit from each transaction? Since advertisers might also potentially benefit from cashback in the form of increased sales, should the burden of providing cashback be the advertiser's or the search engine's? Since advertising slots are sold by auction, the choice of scheme (which includes the ranking and pricing functions for the auction) influences the strategic behavior of advertisers, and therefore the final outcome in terms of the winning advertiser, his payment and the final price to the user. As we will see, these different methods of revenue sharing essentially reduce to creating a means for sellers to price discriminate between online and offline consumer segments (or different online consumer segments): the difference in outcomes arises due to differences in the nature and extent of price discrimination allowed by these revenue-sharing schemes.

The analysis, while technically straightforward, leads to some interesting results, even for the simplest case of an auction for a single slot. First, search engines may earn higher advertising revenue when sharing part of that revenue with consumers rather than keeping all revenue to themselves, even ignoring the effect of the policy on overall user growth. (This is because providing cash back to consumers can increase their likelihood of purchasing items, thereby increasing the probability of an advertising payment.)

However, whether, and how much, revenue increases depends strongly on the scheme used for implementing cashback *as a function* of the input: that is, given a particular set of input values (user parameters and advertiser prices), one scheme can lead to an increase in revenue for the search engine, while the others may not. Further, while one might expect an equivalence between cashback being provided by the search engine and the advertiser (since advertisers can choose their bids strategically in the auction), this is true when the cashback is a fraction of the search engine's profit but *not* when it is a fraction of the item price. Finally, the effect on advertiser or user welfare is also not obvious: depending on the particular scheme being used, it is possible to construct examples where the final, effective, price faced by the user might actually increase with cashback, owing to increased competition amongst advertisers. Thus the problem of cashback is not a straightforward one, and none of these schemes always dominates the others: understanding the marketplace and target user population is essential for effective implementation of revenue sharing.

Related work: The most relevant prior research is that of Jain ([**?**]), making the case that search engines should share the surplus generated by online advertising with users. In contrast, we take a completely neutral approach to revenue-sharing, and provide a model for analyzing its effects on search engine revenue, and user and advertiser welfare.

In some advertising systems, a portion of advertisers' payments go to consumers in the form of coupons, cash back incentives, or membership rewards, either directly from the advertiser or indirectly through an affiliate marketer or other third party lead generator. Several large online affiliate marketing aggregators, for example ebates.com, mypoints.com, and jellyfish.com, function this way, collecting from advertisers on every sale and allocating a portion of their revenue back to the consumer. The main distinction in our work is that the cashback mechanism is embedded in an auction model: advertisers are competing for a sales channel, and the search engine's revenue is determined by the ranking and pricing function used, as well as by the discount offered. We build on the work on equilibrium in sponsored search auctions [**?**,**?**].

Goel et al. [**?**] explore revenue sharing in a ranking or reputation system, describing an ingenious method to incentivize users to fix an incorrect ranking. There is a large body of empirical work on the effect of price discounts and sales on purchases of items, and the impact of different methods of specifying the discount; see, for example, [**?**,**?**]. Researchers have examined consumers' perceptions of search and shopping intentions, at different levels of discounts across two shopping enviroments, one online and the other offline, showing that the shopping intention of the consumers differ at varying discount levels in the two environments [**?**].

2 Model

We model the simplest instance of revenue sharing, where n sellers, each selling an item with posted price p_i, compete for a *single* advertising slot in a cost-per-conversion system (*i.e.*, the winning advertiser makes a payment only when a user buys the item). The search engine, which auctions off this ad slot amongst the sellers, controls the ranking and pricing functions for the auction, and can choose whether and how to include cashback in the mechanism. The key element in our model capturing the effect of revenue sharing is a *price-dependent conversion probability*, $g_i(p)$, which is a decreasing function of p: this introduces a trade-off since decreasing the final price to the user increases the *probability* of a conversion, which may lead to higher expected revenue. This conversion probability function is derived from the following user model: a user is a rational buyer, whose value for item i, v_i, is drawn i.i.d. from a distribution with CDF $F_i(v_i)$. The user buys the item only if the price $p_i \leq v_i$, which has probability $1 - F_i(p)$. Since the user's probability of purchasing item i need not solely be determined by price (it might depend, for instance, on the reputation of seller i, or the relevance of product i to the user), we introduce a price-independent multiplier x_i ($0 < x_i \leq 1$). Thus, the final probability of purchase given price p is $g_i(p) = x_i\,(1 - F(p))$, which is a decreasing function of p. [3]

Associated with seller i, in addition to the posted price p_i and the conversion probability (function) $g_i(p)$, is a production cost c_i, so that a seller's net profit when he sells an item at a price p is $p - c_i$. We assume that posted prices p_i's and the functions g_i are common knowledge to both the search engine and advertisers (this assumption is discussed later); the costs c_i are private to the advertisers. We investigate the trade-off between cashback and expected revenue to the search engine in a single auction; we clarify again that we do not model and study long-term effects of cashback on search engine revenues in this paper.

3 Schemes for Revenue Sharing

We describe and analyze four variants of natural revenue-sharing schemes that the search engine could use when selling a single advertising slot through an auction. For each scheme, we analyze the equilibrium behavior of advertisers, and where possible, state the conditions under which cashback leads to an increase in revenue for the search engine. (Our focus is on search engine revenue since decrease in revenue is the primary argument for a search engine against implementing cashback.) Finally, we compare the schemes against each other. Due to space constraints, all proofs and examples can be found in the full version of this paper.

3.1 Cashback as a fraction of posted price

Specifying cashback as a fraction of the posted price of an item is most meaningful to the user, since he can now compute the exact final price of an item. We consider three natural variants, and specify their equilibria, in order to perform a revenue comparison. Note that the ranking, and therefore the winning advertiser and welfares, are a function of the variants and can also depend on the cashback fraction.

1. *Cashback as a fixed fraction of posted price paid by advertiser.*

[3] For example, if the density f_i is uniform on $[0, W_i]$, $g_i(p) = x_i(1 - \frac{1}{W_i}p)$ is a linear function; if f_i is exponential with parameter λ_i, the resulting g function is exponential as well.

We first consider the scheme where the auction mechanism also dictates the winning advertiser to pay a fixed fraction α of its posted price as cashback to users for every conversion. The fraction α is determined by the search engine ahead of time and is known to all advertisers. In such an auction, advertisers submit a bid b_i which is the maximum amount they are willing to pay the search engine per conversion. The search engine ranks advertisers by expected value per conversion *including* the effect of cashback on conversion probability, *i.e.*, by $g_i(p_i - \alpha p_i)b_i$ (note b_i is the bid and p_i is the posted price). For every conversion, the winning advertiser must pay the search engine the minimum amount he would need to bid to still win the auction; he also pays the cashback to the consumer.
In such an auction, an advertiser's dominant strategy is to bid so that his maximum payment to the search engine plus the revenue share to the user equals his profit, in order to maximize his chance of winning the slot. The following describes the equilibrium of the auction.

Proposition 1 *(Equilibrium behavior) At the dominant strategy equilibrium, advertisers bid* $b_i^{\mathrm{I}} = \max(0, (1-\alpha)\,p_i - c_i)$ *and are ranked by the mechanism according to* $z_i^{\mathrm{I}} = \max\left(0, g_i\left(p_i\,(1-\alpha)\right)\left((1-\alpha)p_i - c_i\right)\right)$*. Let* σ_{I} *be the ranking of advertisers. The winning advertiser,* $\sigma_{\mathrm{I}}\{1\}$ *pays*

$$p_c^{\mathrm{I}} = \frac{z^{\mathrm{I}}_{\sigma_{\mathrm{I}}\{2\}}}{g_{\sigma_{\mathrm{I}}\{1\}}\left(p_{\sigma_{\mathrm{I}}\{1\}}\,(1-\alpha)\right)} \tag{1}$$

for every conversion. The search engine's expected revenue equals the second highest expected value after cashback,

$$R^{\mathrm{I}} = g_{\sigma_{\mathrm{I}}\{1\}}\left(p_{\sigma_{\mathrm{I}}\{1\}}\,(1-\alpha)\right) p_c^{\mathrm{I}} = z^{\mathrm{I}}_{\sigma_{\mathrm{I}}\{2\}}. \tag{2}$$

Note that the ranking σ_{I} is a function of α. For different values of α, different advertisers may win the auction and the advertisers' bids also change.
Even though it is the advertiser who pays the cash-back, it is not always beneficial for the search engine to choose a non-zero fractional cashback, *i.e.*, $\alpha > 0$. We present some sufficient conditions for cashback to be (or not to be) revenue-improving in this case.

Theorem 1. *Suppose* g_i *is such that* $(p - c_i)g_i(p)$ *is continuous and differentiable with respect to* p*, and has a unique maximum at some price* p_i^**. Let* σ_0 *be the ranking of advertisers when there is no cash-back. If* $(p_{\sigma_0\{1\}} - c_{\sigma_0\{1\}})g_{\sigma_0\{1\}}(p_{\sigma_0\{1\}}) > (p_{\sigma_0\{2\}} - c_{\sigma_0\{2\}})g_{\sigma_0\{2\}}(p_{\sigma_0\{2\}})$ *and* $p_{\sigma_0\{2\}} > p_2^*$*, there exists* $\alpha > 0$ *that increases the search engine's revenue. Conversely, if all advertisers' posted prices satisfy* $p_i \leq p_i^*$*, revenue is maximized by setting* $\alpha = 0$*.*

Theorem 1 implies that cash-back may be beneficial to the search engine when the original product prices are "too high", i.e. higher than the optimal prices. The natural question to ask is why any advertiser would want to set a price higher than his optimal price. This relates to the assumption that each advertiser keeps a universal price across all markets (buyer segments or sales channels). Buyers in each market can have a different price sensitivity function g_i. Thus, the universal price can be the optimal price in other markets but higher than the optimal price in the market that the advertiser attempts to reach through the search engine. (It is possible, for instance, that shoppers typically look for deals online, or would want to pay lower prices online than in stores due to uncertainty in product quality or condition.) Example 1 in Appendix B in the full version of the paper illustrates the increase of expected revenue for search engine by choosing a positive α.

2. *Search engine pays cashback as a fixed fraction of posted price.*
Next we consider the scheme where the search engine pays a fixed fraction β of the winning advertiser's posted price as cashback for every conversion. β is determined by the search engine and is known to all advertisers. Naturally, the search engine will only choose values of β so that p_c, the payment per conversion received by the search engine, is greater than or equal to βp_i. Advertisers submit bids b_i. The search engine ranks advertisers by their final (post-cash-back) conversion rate multiplied by their bid, i.e., $g_i(p_i - \beta p_i)b_i$. An advertiser's dominant strategy is to bid so as to maximize his chances of winning the slot without incurring loss:

Proposition 2 *(Equilibrium behavior) Advertisers bid $b_i^{\mathrm{II}} = p_i - c_i$ and are ranked by $z_i^{\mathrm{II}} = g_i\left(p_i\left(1-\beta\right)\right)\left(p_i - c_i\right)$ at the dominant strategy equilibrium. Let σ_{II} be the ranking of advertisers. The winning advertiser, $\sigma_{\mathrm{II}}\{1\}$, pays*

$$p_c^{\mathrm{II}} = \frac{z_{\sigma_{\mathrm{II}}\{2\}}^{\mathrm{II}}}{g_{\sigma_{\mathrm{II}}\{1\}}\left(p_{\sigma_{\mathrm{II}}\{1\}}\left(1-\beta\right)\right)}. \tag{3}$$

for every conversion. The search engine's expected revenue is

$$\begin{aligned} R^{\mathrm{II}} &= g_{\sigma_{\mathrm{II}}\{1\}}\left(p_{\sigma_{\mathrm{II}}\{1\}}\left(1-\beta\right)\right)\left(p_c^{\mathrm{II}} - \beta p_{\sigma_{\mathrm{II}}\{1\}}\right) \\ &= z_{\sigma_{\mathrm{II}}\{2\}}^{\mathrm{II}} - \beta p_{\sigma_{\mathrm{II}}\{1\}} g_{\sigma_{\mathrm{II}}\{1\}}\left(p_{\sigma_{\mathrm{II}}\{1\}}\left(1-\beta\right)\right). \end{aligned} \tag{4}$$

In this case also, the search engine may increase its expected revenue when using this scheme. Suppose $g_i(p_i) = 1 - 0.1p_i$. Three advertisers A, B, and C participate in the auction. They have prices $p_A = 6$, $p_B = 9$, and $p_C = 10$ respectively; $c = 0$ for all advertisers. Then, by setting $\beta = 0.4737$ the search engine increases its expected revenue from 0.9 to 2.4931 and the final price faced by the user drops from 6 to 4.74.

3. *Advertiser chooses amount of cashback and pays it.*
More expressiveness is provided to the advertisers if they are allowed to bid both on the fractional discount they offer, as well as their per-conversion payment to the search engine. Both of these are then used in the ranking function. The search engine runs an auction that does not specify the fraction of revenue share required. Instead, the auction rule requires the advertiser to

submit both a bid b_i and a fraction γ_i ($0 \leq \gamma_i \leq 1$). Advertisers are ranked by conversion rate (including cashback) multiplied by bid, i.e. $g_i\left(p_i(1-\gamma_i)\right) b_i$. The payment of the winning advertiser is as follows: his net payment is $\gamma_i p_i + p_c$, where p_c is the minimum amount he needs to bid, keeping γ_i fixed, to win the auction. The dominant strategy for all advertisers is to choose γ_i to maximize their values, and for the choice of γ_i, to bid their true value after the effect of cashback.

Proposition 3 *At the dominant strategy equilibrium, advertisers select* $\gamma_i^* = \arg\max_{0 \leq \gamma_i \leq 1} x_i g_i\left((1-\gamma_i)\, p_i\right)\left((1-\gamma_i)p_i - c_i\right)$*, bid* $b_i^{\mathrm{III}} = (1-\gamma_i^*)\, p_i - c_i$*, and are ranked by* $z_i^{\mathrm{III}} = g_i\left(p_i\left(1-\gamma_i^*\right)\right)\left((1-\gamma_i^*)p_i - c_i\right)$*. Let* σ_{III} *be the ranking of advertisers. The winning advertiser,* $\sigma_{\mathrm{III}}\{1\}$*, pays the search engine*

$$p_c^{\mathrm{III}} = \frac{z^{\mathrm{III}}_{\sigma_{\mathrm{III}}\{2\}}}{g_{\sigma_{\mathrm{III}}\{1\}}\left(p_{\sigma_{\mathrm{III}}\{1\}}\left(1-\gamma^*_{\sigma_{\mathrm{III}}\{1\}}\right)\right)} \tag{5}$$

and pays the user $\gamma^*_{\sigma_{\mathrm{III}}\{1\}} p_{\sigma_{\mathrm{III}}\{1\}}$ *per conversion. The search engine's expected revenue is*

$$R^{\mathrm{III}} = g_{\sigma_{\mathrm{III}}\{1\}}\left(p_{\sigma_{\mathrm{III}}\{1\}}\left(1-\gamma^*_{\sigma_{\mathrm{III}}\{1\}}\right)\right) p_c^{\mathrm{III}} = z^{\mathrm{III}}_{\sigma_{\mathrm{III}}\{2\}}. \tag{6}$$

Note that allowing the advertiser to choose γ_i as well as b_i essentially allows them to choose an effective new "price". Consequently, if possible the advertiser selects γ_i so that the new price equals his optimal price. For $p_i > p_i^*$, this γ_i^* is such that $(1-\gamma_i^*)p_i = p_i^*$, where p_i^* is the price that maximizes the function $(p - c_i)g_i(p)$. The following theorem shows that in this scheme, the search engine's expected revenue is always weakly larger than without cashback.

Theorem 2. *Let* R^0 *denote search engine's expected revenue without cashback. For the same set of advertisers,* $R^{III} \geq R^0$*.*

Example 2, Appendix B in the full version of this paper illustrates the increase of search engine's expected revenue with this scheme.

3.2 Cashback as a fraction of search engine revenue

Another natural way to specify a revenue share is to describe it as a fraction α of the search engine's revenue, *i.e.*, the payment per conversion; this corresponds to the search engine sharing its surplus with the user, who is an essential component of the revenue generation process. Unless the search engine charges a fixed price per conversion, it is hard to include post-cashback conversion rates to determine the ranking, since the amount of cashback depends on the ranking. Thus, we use the conversion rate before cashback to rank advertisers. In this scheme, advertisers are ranked according to $g_i(p_i)b_i$, where b_i is the per-conversion bid submitted by advertiser i, and search engine pays a fixed fraction δ of its revenue per conversion as cashback. Again, it is a dominant strategy for advertisers to bid their true value:

Proposition 4 *Advertisers bid $b_i^{\mathrm{IV}} = p_i - c_i$ and are ranked by $z_i^{\mathrm{IV}} = g_i(p_i)(p_i - c_i)$ at the dominant strategy equilibrium. Let σ_{IV} be the ranking of advertisers. The winning advertiser, $\sigma_{\mathrm{IV}}\{1\}$, pays*

$$p_c^{\mathrm{IV}} = \frac{g_{\sigma_{\mathrm{IV}}\{2\}}(p_{\sigma_{\mathrm{IV}}\{2\}})(p_{\sigma_{\mathrm{IV}}\{2\}} - c_{\sigma_{\mathrm{IV}}\{2\}})}{g_{\sigma_{\mathrm{IV}}\{1\}}(p_{\sigma_{\mathrm{IV}}\{1\}})} \tag{7}$$

per conversion. The revenue of the search engine with cashback is

$$R^{\mathrm{IV}} = g_{\sigma_{\mathrm{IV}}\{1\}}(p_{\sigma_{\mathrm{IV}}\{1\}} - \delta p_c^{\mathrm{IV}})\frac{z_{\sigma_{\mathrm{IV}}\{2\}}}{g_{\sigma_{\mathrm{IV}}\{1\}}(p_{\sigma_{\mathrm{IV}}\{1\}})} p_c^{\mathrm{IV}}(1-\delta). \tag{8}$$

Note that this ranking is *independent* of the value of δ, the cashback fraction: σ_{IV} is the same as σ_0, the ranking without cashback.

It is also possible to request the advertiser to pay the cashback that is specified as a fixed fraction of the search engine's revenue. We show that it is equivalent to the case that the search engine pays the cashback.

Theorem 3. *The scheme where search engine pays δ fraction of its revenue per conversion as cashback is equivalent to the scheme where the advertiser pays $\delta/(1-\delta)$ fraction of the search engine's revenue per conversion as cashback, regarding to the utilities of the user, the advertisers, and the search engine.*

Note that when revenue share is specified as a fraction of search engine revenue, the search engine may choose the optimal fraction δ *after* advertisers submit their bids. This will not change the equilibrium bidding behavior of advertisers, in contrast to the case where advertisers pay the cashback. Since the optimal cashback δ might be 0, choosing δ after collecting bids ensures that the search engine's revenue never decreases because of cashback.

Whether or not the search engine can increase its revenue by giving cash-back depends on the posted prices of the top two advertisers and their g functions.

Theorem 4. *If there exists $\delta > 0$ such that $g_{\sigma_{\mathrm{IV}}\{1\}}(p_{\sigma_{\mathrm{IV}}\{1\}} - \alpha p_c)(1-\delta) \geq g_{\sigma_{\mathrm{IV}}\{1\}}(p_{\sigma_{\mathrm{IV}}\{1\}})$, revenue sharing with parameter δ increases the expected revenue of the search engine. For linear $g_i = x_i(1-kp_i)$ and $c_i = 0$, $\delta > 0$ when $p_{\sigma_{\mathrm{IV}}\{1\}} + p_c^{\mathrm{IV}} > 1/k$.*

3.3 Comparison between schemes

The first three schemes described above all specify revenue share as a fraction of posted price, while the fourth scheme specifies revenue share as a fraction of the search engine revenue. The following results characterize the choice of mechanism to maximize the search engine's revenue, when revenue share is expressed as a fraction of posted price.

Theorem 5. *Given a set of advertisers, $R^{\mathrm{III}} \geq R^{\mathrm{I}}$ for all α.*

Theorem 6. *Given a set of advertisers, $R^{\mathrm{I}} \geq R^{\mathrm{II}}$ if $\alpha = \beta$ and the ranking according to $p_i * g(p_i(1-\beta))$ is the same as the ranking according to $(p_i - c_i) * g(p_i(1-\beta))$.*

This gives us a result on maximizing revenue when cashback is specified as a fraction of the posted prices for the special cases below.

Corollary 1 *When $c_i = 0$, or $c_i = \mu p_i$ for all i, $R^{\mathrm{III}} \geq R^{\mathrm{I}} \geq R^{\mathrm{II}}$. Thus revenue is maximized when the search engine allows advertisers to choose and pay the fraction γ_i of their posted prices.*

When revenue share is expressed as a fraction of the posted price, allowing advertisers to choose the fraction of revenue share (the third scheme) can lead to the highest revenue for the search engine in many cases. Thus, we compare it with the case when revenue share is specified as a fraction of the advertising revenue (the fourth scheme). We have the following result.

Proposition 5 *Neither the revenue-maximizing cashback scheme with cashback as a fraction of posted price, nor the revenue-maximizing scheme with cashback as a fraction of search engine revenue, always dominates the other in terms of generating higher expected revenue for the search engine.*

Thus, depending on the set of posted prices, the expected revenue of the search engine in either the third or the fourth scheme can be higher. Both schemes, however, are always weakly revenue improving: in the third scheme where advertisers specify the cashback amount, the search engine needs to make no choice and, according to Theorem 2, the search engine's revenue is at least as large as that without cashback. In the fourth scheme also, the search engine can choose the optimal fraction after the bids have been submitted, ensuring that cashback never leads to loss in revenue.

We note that whether cashback can increase search engine revenue or not also depends on the revenue sharing schemes. Given a set of advertiser prices, it is possible that one scheme can increase the revenue of search engine by providing positive cashback, while the other scheme is better off not giving cashback at all. Examples 3 and 4 in Appendix B in the full version of the paper support this with two specific instances.

4 Conclusion

We model revenue sharing with users in the context of online advertising auctions in a cost-per-conversion system, in which the winning advertiser pays the search engine only in the event of a conversion. The conversion probability of a user is modeled as a decreasing function of the final product price that the user faces. Thus, sharing revenue with the user may increase the conversion probability sufficiently to lead to a short-term increase in the search engine's expected revenue, despite the fact that the per-conversion revenue decreases.

We study four schemes for a search engine to specify the revenue share in the auction setting. When the revenue share is expressed as a fraction of the winning advertiser's posted price, we have (1) advertiser pays cashback as a fixed fraction of posted price; (2) search engine pays cashback as a fixed fraction of posted price; and (3) advertiser determines and pays cashback. If the revenue share is specified as a fraction of the advertiser's revenue per conversion, we consider (4) the search engine pays cashback as a fixed fraction of its revenue. We analyze the equilibrium of the auction for the four schemes and show that for all four schemes there are situations in which search engine can increase its short-term expected revenue by allowing revenue sharing. Scheme (3) dominates scheme (1) and (2) in many situations in terms of maximizing search engine revenue. However, neither scheme (3) nor scheme (4) are universally better for generating higher search engine revenue. We note that although revenue sharing often leads to lower final prices to users, this need not always be the case: there exist advertiser prices under which the revenue maximizing cashback fraction leads to increased final price to the user, as shown in Example 1, Appendix B in the full version of this paper.

The properties of these revenue sharing mechanisms rely strongly on the assumption that advertisers keep a universal price across all sales channels, which is often the case in reality. If advertisers can or are willing to charge channel-specific-prices, they will select an optimal price to participate in the advertising auction. In return, the search engine no longer needs to, or will not find it profitable to share revenue with the user. In fact, revenue sharing with users is an indirect way, controlled by the search engine, to achieve price discriminations across different sales channels.

Appendix

A Proofs

A.1 Proof of Theorem 1

First, consider the case that $p_{\sigma_0\{1\}} > p_1^*$. When $p_{\sigma_0\{1\}} > p_1^*$ and $p_{\sigma_0\{2\}} > p_2^*$, there exists some $\alpha > 0$ such that $p_{\sigma_0\{1\}}(1-\alpha) > p_1^*$ and $p_{\sigma_0\{2\}}(1-\alpha) > p_2^*$. Since $(p - c_i)g_i(p)$ increases while p decreases in the range of $p > p_i^*$, expected values of both advertisers increase with such α. The search engine's revenue, which is the second highest expected value among all advertisers, is greater or equal to the expected value of advertiser $\sigma_0\{2\}$ under the new ranking, which is higher than that of the no cash-back case. Thus there exists some $\alpha > 0$ such that search engine's revenue increases.

Next consider the case $p_{\sigma_0\{1\}} \leq p_1^*$. Since $(p_{\sigma_0\{1\}} - c_{\sigma_0\{1\}})g_{\sigma_0\{1\}}(p_{\sigma_0\{1\}}) > (p_{\sigma_0\{2\}} - c_{\sigma_0\{1\}})g_{\sigma_0\{2\}}(p_{\sigma_0\{2\}})$, there exists some $\alpha > 0$ such that

$$(p_{\sigma_0\{1\}}(1-\alpha) - c_{\sigma_0\{1\}})g_{\sigma_0\{1\}}(p_{\sigma_0\{1\}}(1-\alpha)) > (p_{\sigma_0\{2\}} - c_{\sigma_0\{2\}})g_{\sigma_0\{2\}}(p_{\sigma_0\{2\}}),$$

and $p_{\sigma_0\{2\}}(1-\alpha) > p_2^*$, by the continuity of p and $(p - c_i)g_i(p)$. While the ranking might change, the second highest expected value among all advertisers is greater or equal to the smaller of $(p_{\sigma_0\{1\}}(1-\alpha) - c_{\sigma_0\{1\}})g_{\sigma_0\{1\}}(p_{\sigma_0\{1\}}(1-\alpha))$ and $(p_{\sigma_0\{2\}}(1-\alpha) - c_{\sigma_0\{2\}})g_{\sigma_0\{2\}}(p_{\sigma_0\{2\}}(1-\alpha))$, which is higher than $p_{\sigma_0\{2\}}g_{\sigma_0\{2\}}(p_{\sigma_0\{2\}})$. Thus the search engine's revenue increases for this nonzero α. Finally, if $p_i \leq p_i^*$ for all advertisers, $(p - c_i)g_i(p)$ decreases when p decreases in the range of $p_i \leq p_i^*$. The expected values of all advertisers decrease, including the second highest expected value. Hence, the search engine is better off by setting $\alpha = 0$.

A.2 Proof of Theorem 2

For a fixed advertiser i, the expected value is always at least as high when she chooses the discount factor γ_i herself as compared to no cashback:

$$z_i^{\mathrm{III}} = \max_{\gamma_i} g_i(p_i(1-\gamma_i))(p_i(1-\gamma_i) - c_i) \geq g_i(p_i)(p_i - c_i) = z_i^0,$$

for every $0 \leq \alpha \leq 1$. Since the revenues in both cases are the second highest expected values, we have

$$R^0 = z^0_{\sigma_0\{2\}} \leq z^{\mathrm{III}}_{\sigma_0\{2\}} \leq z^{\mathrm{III}}_{\sigma_{\mathrm{III}}\{2\}} = R^{\mathrm{III}}.$$

A.3 Proof of Theorem 3

Suppose the advertiser is required to pay a fixed fraction θ of the search engine's revenue as cash back. The dominant strategy for all advertisers is to bid $b_i^V = \frac{p_i - c_i}{1+\theta}$. Advertiser i's expected value before cash-back, denoted as z_i^{IV} is

$$z_i^{\mathrm{IV}} = \frac{g_i(p_i)(p_i - c_i)}{1+\theta}.$$

It is easy to see that the ranking of advertisers in this mechanisms is the same as the ranking when the search engine pays a fixed fraction δ of its revenue as cash back, i.e., $\sigma_{\mathrm{V}} = \sigma_{\mathrm{IV}}$. Thus, advertiser $\sigma_{\mathrm{IV}}\{1\}$ is the winner and pays

$$p_c^{\mathrm{V}} = \frac{g_{\sigma_{\mathrm{IV}}\{2\}}(p_{\sigma_{\mathrm{IV}}\{2\}})(p_{\sigma_{\mathrm{IV}}\{2\}} - c_{\sigma_{\mathrm{IV}}\{2\}})}{(1+\theta)g_{\sigma_{\mathrm{IV}}\{1\}}(p_{\sigma_{\mathrm{IV}}\{1\}})} = \frac{p_c^{\mathrm{IV}}}{1+\theta} \tag{9}$$

The final price to the user is

$$p' = p_{\sigma_{\mathrm{IV}}\{1\}} - \frac{x_{\sigma_{\mathrm{IV}}\{2\}} g(p_{\sigma_{\mathrm{IV}}\{2\}})(p_{\sigma_{\mathrm{IV}}\{2\}} - c_{\sigma_{\mathrm{IV}}\{2\}})}{(1+\theta)x_{\sigma_{\mathrm{IV}}\{1\}} g(p_{\sigma_{\mathrm{IV}}\{1\}})} = p_{\sigma_{\mathrm{IV}}\{1\}} - \frac{\theta}{1+\theta} p_c^{\mathrm{IV}}.$$

The revenue to the search engine is

$$R_c^{\mathrm{V}} = x_{\sigma_{\mathrm{IV}}\{1\}} g(p_{\sigma_{\mathrm{IV}}\{1\}} - \frac{\theta}{1+\theta} p_c^{\mathrm{IV}}) \frac{p_c^{\mathrm{IV}}}{1+\theta}. \tag{10}$$

When $\theta = \delta/(1-\delta)$, both the final prices to the user and the search engine revenue are the same for the two cases.

A.4 Proof of Theorem 4

Note that since the ranking does not change, and the pricing scheme is as given, the per-conversion payment does not change with δ. The revenue of the search engine with cashback is

$$R^{\mathrm{IV}} = g_{\sigma_{\mathrm{IV}}\{1\}}(p_{\sigma_{\mathrm{IV}}\{1\}} - \delta p_c^{\mathrm{IV}}) \frac{z_{\sigma_{\mathrm{IV}}\{2\}}}{g_{\sigma_{\mathrm{IV}}\{1\}}(p_{\sigma_{\mathrm{IV}}\{1\}})} p_c^{\mathrm{IV}}(1-\delta).$$

The revenue of the search engine without cashback is obtained by setting $\delta = 0$ in the above expression. Thus, when there exists $\delta > 0$ such that

$$g_{\sigma_{\mathrm{IV}}\{1\}}(p_{\sigma_{\mathrm{IV}}\{1\}} - \delta p_c)(1-\delta) \geq g_{\sigma_{\mathrm{IV}}\{1\}}(p_{\sigma_{\mathrm{IV}}\{1\}}),$$

cashback can increase revenue of the search engine.

When functions g are linear, $g_i = x_i(1 - kp_i)$ where $0 < x_i \leq 1$ and $k > 0$, and $c_i = 0$, the search engine's revenue without cash-back is

$$R^0 = x_{\sigma_{\mathrm{IV}}\{1\}}(1 - kp_{\sigma_{\mathrm{IV}}\{1\}})p_c^{\mathrm{IV}} = x_{\sigma_{\mathrm{IV}}\{2\}}(1 - kp_{\sigma_{\mathrm{IV}}\{2\}})p_{\sigma_{\mathrm{IV}}\{2\}}.$$

Its revenue with cash-back is

$$R^{\mathrm{IV}} = x_{\sigma_{\mathrm{IV}}\{1\}}(1 - kp_{\sigma_{\mathrm{IV}}\{1\}} + k\delta p_c^{\mathrm{IV}})(1-\delta)p_c^{\mathrm{IV}}. \tag{11}$$

In order for search engine to be better off giving cash-back, we need that

$$x_{\sigma_{\mathrm{IV}}\{1\}} k\delta\,(1-\delta)\left(p_c^{\mathrm{IV}}\right)^2 - \delta x_{\sigma_{\mathrm{IV}}\{2\}}(1 - kp_{\sigma_{\mathrm{IV}}\{2\}})p_{\sigma_{\mathrm{IV}}\{2\}} \geq 0,$$

which gives that

$$\begin{aligned} 1 - \delta &\geq \frac{x_{\sigma_{\mathrm{IV}}\{1\}}\left(1 - kp_{\sigma_{\mathrm{IV}}\{1\}}\right)^2}{kx_{\sigma_{\mathrm{IV}}\{2\}}\left(1 - kp_{\sigma_{\mathrm{IV}}\{2\}}\right)p_{\sigma_{\mathrm{IV}}\{2\}}} = \frac{(1 - kp_{\sigma_{\mathrm{IV}}\{1\}})}{kp_c^{\mathrm{IV}}} \\ \Rightarrow \delta &\leq 1 - \frac{(1 - kp_{\sigma_{\mathrm{IV}}\{1\}})}{kp_c^{\mathrm{IV}}}. \end{aligned}$$

When $p_{\sigma_{\mathrm{IV}}\{1\}} + p_c^{\mathrm{IV}} > 1/k$, δ is greater than 0, *i.e.*, revenue sharing with the user actually increases the search engine's expected revenue.

A.5 Proof of Theorem 5

For a fixed advertiser i, the expected value is always at least as high when she chooses the discount factor γ_i herself as compared to when the search engine chooses α:

$$z_i^{\mathrm{III}} = \max_{\gamma_i} g_i(p_i(1-\gamma_i))(p_i(1-\gamma_i) - c_i) \geq g_i(p_i(1-\alpha))(p_i(1-\alpha) - c_i) = z_i^{\mathrm{I}},$$

for every $0 \leq \alpha \leq 1$. Since the revenues in both cases are the second highest expected values, we have

$$R^{\mathrm{I}} = z^{\mathrm{I}}_{\sigma_{\mathrm{I}}\{2\}} \leq z^{\mathrm{III}}_{\sigma_{\mathrm{I}}\{2\}} \leq z^{\mathrm{III}}_{\sigma_{\mathrm{III}}\{2\}} = R^{\mathrm{III}}.$$

A.6 Proof of Theorem 6

If the conditions are satisfied, it can be seen that the rankings for the two schemes, which are according to $g_i(p_i(1-\beta))(p_i - c_i)$, and $g_i(p_i(1-\beta))(p_i(1-\beta) - c_i)$ are the same. Specifically, the top two advertisers are the same. We have

$$\begin{aligned} R^{\mathrm{II}} &= g_2(p_2(1-\beta))(p_2 - c_2) - \beta p_1 g_1(p_1(1-\beta)) \\ &\leq g_2(p_2(1-\beta))(p_2 - c_2) - \beta p_2 g_2(p_2(1-\beta)) \\ &= g_2(p_2(1-\beta))(p_2(1-\beta) - c_2) = R^{\mathrm{I}}, \end{aligned}$$

where we used the condition on the rankings in the second line, and the fact that the rankings are identical in the final step.

A.7 Proof of Proposition 5

We use two examples to prove the proposition. Case 1 describes a situation where the optimal revenue obtained scheme II is larger than the maximum possible revenue generated by scheme II.

Case 1: Suppose $p_i = x_i(1 - 0.1p_i)$. There are 3 advertisers A, B, and D, who have $p_A = 6$, $p_B = 8$, $p_D = 7$, and $x_A = 0.7$, $x_B = 0.3$, $x_D = 0.9$. With advertisers bidding on both revenue sharing fractions and payment to search engine, the second highest expected value is from bidder A, with a value of 1.75, which is the expected revenue to the search engine. When search engine pays a fixed fraction of his profit as cashback, the ranking of the advertisers does not change with the fraction of cash back. The revenue to the search engine is maximized at

$$\delta^* = \max(0, \frac{1}{2}(1 - \frac{1 - 0.1p_A}{0.1p_c})) = 0.26.$$

At this value of α, the search engine's revenue is

$$R = g_D(p_D - \delta^* p_c) * (p_c - \delta^* p_c) = 1.91.$$

So the optimal expected revenue of the search engine is higher than that with the former case. For comparison, the expected revenue of the search engine with no cash-back at all is 1.68.

The example in the other direction is not too surprising.

Case 2: Suppose $p_i = x_i(1 - 0.1p_i)$. There are 3 advertisers A, B, and D, who have $p_A = 8$, $p_B = 7$, $p_D = 6$, and $x_A = 0.8$, $x_B = 0.7$, $x_D = 0.3$. With advertisers bidding on both revenue sharing fractions and payment to search engine, the second highest expected value is from bidder B, with a value of 1.75, which is the search engine's expected revenue. When search engine pays a fixed fraction of his profit as cashback, the ranking of the advertisers does not change with δ, and the revenue to the search engine is maximized at

$$\delta^* = \max(0, \frac{1}{2}(1 - \frac{1 - 0.1p_A}{0.1p_c})) = 0.25.$$

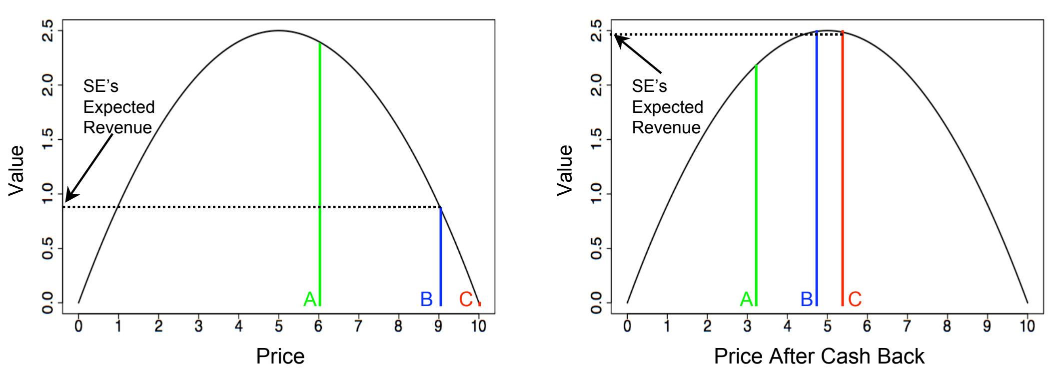
At this value of δ, the search engine's expected revenue is

$$R = g_A(p_A - \delta^* p_c) * (p_c - \delta^* p_c) = 1.45.$$

So the optimal expected revenue of the search engine in the later case is less than that in the former case. For comparison, the expected revenue of the search engine revenue with no cash-back at all is 1.28.

B Examples

Example 1 *Suppose $g_i(p_i) = 1 - 0.1p_i$. There are three advertisers A, B, and C competing for one advertising slot. Their prices are $p_A = 6$, $p_B = 9$, and $p_C = 10$; $c = 0$ for all advertisers. Figure 1(a) plots the expected values of advertisers, z_i^{I}, when there is no cash-back, i.e. $\alpha = 0$. Advertiser A has the highest value, followed by advertisers B and C. The search engine's expected revenue equals the second highest value, $R^{\mathrm{I}} = z_B^{\mathrm{I}} = 0.9$. The revenue optimal α for the search engine is the one such that $p_{\sigma_{\mathrm{I}}\{2\}}$ is as close to $1/(2k_i)$ as possible, i.e. $z^{\mathrm{I}}_{\sigma_{\mathrm{I}}\{2\}}$ is maximized. Figure 1(b) plots the expected values of advertisers when the search engine selects the optimal $\alpha = 0.4737$. Prices after cash-back for advertisers A, B, and C are 3.16, 4.74, and 5.26 respectively. Now, advertisers B and C have the highest value, followed by advertiser A. The search engine's expected revenue equals 2.4931, which is much higher than when there is no cash-back. The final price (price after cash-back) decreases from 6 to 4.74, supposing the search engine breaks tie by selecting the advertiser that has a lower price. The user is better off since he faces a lower price. However, it is not always the case that the final price is lower when there is cash back. If advertiser A has price $p_A = 4$, the search engine can still increase its revenue by offering the same percentage of cash-back, but the final price would increases from 4 to 4.74.*



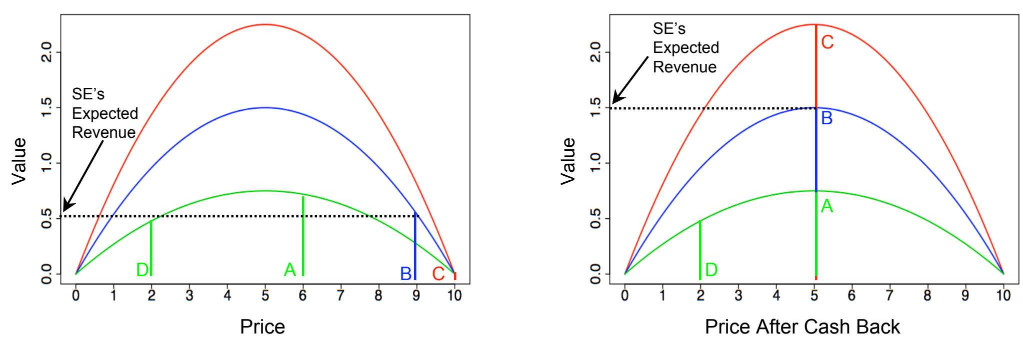
(a) No Cashback (b) Cashback with $\alpha = 0.4737$

Fig. 1. Example 1 – Expected values of advertisers at the Nash Equilibrium

Example 2 *Suppose $g_i(p_i) = x_i(1 - 0.1p_i)$. There are four advertisers A, B, C, and D, with $x_A = 0.3$, $x_B = 0.6$, $x_c = 0.9$, and $x_D = 0.3$.Their prices are $p_A = 6$, $p_B = 9$, $p_C = 10$, and $p_D = 2$; $c = 0$ for all advertisers.*

Let $p_i^f = (1 - \gamma_i)p_i$ denote advertiser i's price after cash-back. Advertiser i's value z_i^{III} is a quadratic function in terms of p_i^f. Figure 2(a) plots advertiser's value z_i^{III} when there is no cash-back, $\gamma_i = 0$. The blue, and red curves are for advertisers B, and C respectively. The green curve is for both advertisers A and D since $x_A = x_D$. Because x_i's are different, three curves scale vertically. We can see that advertiser A has the highest value, followed by advertisers B, C, and D. The search engine's expected revenue equals the second highest expected value, $R^{\mathrm{III}} = z_B^{\mathrm{III}} = 0.54$. If the search engine allows advertisers to select the fraction of revenue share, γ_i, in the auction. Advertiser A, B and C will choose γ_i such that $p_i^f = 5$. Hence, $\gamma_A = 0.17$, $\gamma_B = 0.44$ and $\gamma_C = 0.5$. Advertiser D will still set $\gamma_D = 0$. Figure 2(b) plots the situation when advertisers are allowed to choose γ_i.

Prices after cash-back for advertisers A, B, C, and D are 5, 5, 5, and 2 respectively. Now, advertiser C has the highest value, followed by advertisers B, A, and D. The search engine's expected revenue equals the expected value of advertiser B, which is 1.5, and is higher than that with no cashback.



(a) No Cashback

(b) Cashback with $\gamma_A = 0$, $\gamma_B = 0.44$, and $\gamma_C = 0.5$

Fig. 2. Example 2 – Values of advertisers at the Nash Equilibrium

Example 3 *Suppose we have two advertisers A and B with linear functions, $g_i(p_i) = x_i(1 - 0.1p_i)$. $x_A = 1$; and $x_B = 0.5$. Suppose $p_A = 7$ and $p_B = 5$, i.e.advertiser B posts his optimal price. There is no increase in revenue from the third scheme.*

But in the fourth scheme, $p_c = 4.1667$, and $p_A + p_c$ is greater than 10, which is the condition for existence of cashback in the fourth scheme. At this p_A, advertiser A is still the winner, since his expected value without cashback is $g_A(p_A)p_A = 2.1$, which is greater than 1.25.)

Example 4 *Suppose* $g_i(p_i) = x_i(1 - 0.1p_i)$*, and there are two advertisers* A *and* B*, with* $x_A = x_B = 1$*. Suppose that the posted prices are* $p_A = 5$ *and* $p_B = 6$*. In this case, there is no cashback in the fourth scheme, since* $p_c = 4.8$*, and* $p_A + p_c < 10$*. However, the revenue maximizing cashback from the third scheme gives a revenue of* 2.5*, which is greater than the revenue without cashback,* 2.4*.*