New Mathematics and Natural Computation

PREDICTING UNCERTAIN OUTCOMES USING INFORMATION MARKETS: TRADER BEHAVIOR AND INFORMATION AGGREGATION

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Forecasting seems to be a ubiquitous endeavor in human societies. In this paper, information markets are introduced as a promising mechanism for predicting uncertain outcomes. Information markets are markets that are specially designed for aggregating information and making predictions on future events. A generic model of information markets is proposed. We derive some fundamental properties on when information markets can converge to the direct communications equilibrium, which aggregates all information across traders and is the best possible prediction for the event under consideration. Information markets, if properly designed, have substantial potential to facilitate organizations in making better informed decisions.

Keywords: Information market; Prediction; Trader behavior; Information aggregation.

1. Introduction

Predicting outcomes of relevant uncertain events is a crucial part of decision making processes. For example, companies rely on forecasts of consumer demand, raw material supply, and possible changes in market regulations, to make their production plans and decisions. How to accurately estimate outcomes of various random variables that relate to a decision problem is essential to rational decision making.

For decades, scientists have devoted themselves to developing and exploring various forecasting methods, which can be roughly divided into statistical and non-statistical approaches. Statistical methods, including econometric models, time series analysis, and some machine learning techniques, are based on historical data. Non-statistical methods frequently rely on judgments and opinions of experts. Both approaches have limitations. Statistical methods require not only the existence of enough historical data but also that past data contain valuable information about future events. Eliciting expert opinions means identifying experts, soliciting their participation, and determining how to combine different opinions when experts are not in agreement, which are often not easy[1,2].

Information markets, markets that are specially designed for aggregating information and making predictions about future events, have recently emerged as a promising alternative forecasting tool. Information markets as a forecasting method have many advantages. Compared with statistical forecasting methods, information markets can incorporate real-time information, which is not contained in historical data. Compared with eliciting expert opinions, information markets are less constrained by space and time; they eliminate the efforts of identifying experts and soliciting their participation, and hence are often less expensive in practice; and they do not need to deal with conflicting opinions. More importantly, information markets can potentially make real-time predictions that take advantage of widely dispersed information, which is usually hard to capture using other forecasting methods.

Despite merits of information markets, why they work, how they work, when they work, and how to design effective information markets are still open questions to a large extent. If information markets are to be used to assist businesses, universities, and governments in making critical decisions in the real world, investigating these questions is imperative. Aiming at gaining deeper understanding of information markets as a forecasting tool, in this paper, we examine when information markets can make the "best" predictions through modeling and analysis. By saying the "best", we mean that these predictions take advantage of all information across market participants. Thus, compared with other predictions that are made based on less information, they are the best informed predictions. The results of the paper help to understand theoretical properties of information markets, which are currently lack of attention but are in great need for eventually establishing the underpinnings of information markets.

The remainder of the paper is organized as follows. Section 2 answers why information markets can be used to make predictions. It reviews basics of information markets and related research. In section 3, we outline a generic model of information markets for theoretical analysis. We analyze convergence properties of information markets in Section 4. Section 5 discusses our results and their implications. It also presents real world applications to show where information markets are applicable. Section 6 concludes the study and identifies areas for further research.

2. Background and Related Work

2.1. *Basics of Information Markets*

An information market can be roughly defined as a market mechanism that ties to a random variable and is specially designed for forecasting its value. Depending on the context they are used, information markets are also called prediction markets, forecasting markets, or decision markets. In this paper, we adopt the name information markets for consistency.

The idea of information markets arises from studies of financial markets. The value of traded securities in a financial market is uncertain at the time of trading. Market traders buy or sell securities based on their current information. A subtle property of financial markets, which has been supported by both theoretical and empirical research, is that, under certain conditions, markets can aggregate less-than-perfect, dispersed information across many individual traders, so that prices of a security summarizes all relevant information across traders[3,4,5]. If the security price can incorporate all available information of market traders, it can be viewed as a consensus prediction about the value of the security. This implies that financial market mechanisms can be used in aggregating information and making predictions.

Roughly speaking, setting up an information market means to create a security for a random variable of interest and provide a marketplace for trading the security. If we want to predict the value of a random variable, we can turn it into a security, whose payoff equals the realized value of the random variable, and trade the security in a market. Dispersed information related to the random variable is steadily aggregated and incorporated into the market price, which, at equilibrium, is an approximation of the expectation of the random variable.

The 2004 U.S. presidential vote share market is an example of information markets, which was hosted at the Iowa Electronic Markets (IEM)[a]. The market ties security payoffs to the percentage of the total popular vote received by the candidates in the election. For instance, the security for George W. Bush who receives 51.5% of the popular votes cast pays 51.5 cents per share to its holders. Market participants trade the security based on their expectations about the candidate's vote share. Thus, the price of the security is market participants' prediction of the vote share that George W. Bush will receive.

One of the most important advantages of information markets, as opposed to opinion pooling methods, is that they can make a single consistent prediction. Opinion pooling methods usually attempt to directly aggregate diverse opinions of individual experts, ignoring the information that leads experts to reach their opinions. Although they report a single prediction, this is often not a consensus among experts. Experts still disagree with each other, since the information on which they make their individual predictions can be different. Information markets, on the other hand, enable market participants to update their individual predictions as they be-

[a]http://www.biz.uiowa.edu/iem/

come aware of more information through interacting with the market. Eventually, individual predictions converge to an identical estimation, which is the equilibrium market price. For example, in the above mentioned U.S. presidential vote share market, suppose that the current price of a security for George W. Bush is 50 cents. If some market traders possess crucial information that leads them to predict that George W. Bush will receive 60% vote share, they will buy more securities as long as the price is below 60 cents. Their activities will drive up the market price. The increase of the price signals other traders that some information in the market, which they don't know, forecasts that George W. Bush will receive a higher vote share. They will then probably adjust their own predictions accordingly. The market reaches its equilibrium only when all traders believe that the market price represents their predictions. In addition, traders in information markets arguably have incentives to be honest in contributing what they know in the trading activities, because they must "put their money where their mouth is" [6,7].

Since most uncertain future events can be viewed as random variables, information markets can function as a powerful tool for combining individual opinions and generating aggregated forecasts about future events. Information markets have been proved effective in many domains, including politics[8,9,10], entertainment[11], and sports[12,13]. Implemented properly, they have the potential to assist businesses, universities, and governments making critical decisions.

2.2. *Related Work*

Research of information markets usually takes one of the three approaches: experimental studies, analysis of real world online game markets, and theoretical modeling of information markets. Most of the work supports that information markets are very effective at aggregating information and making predictions.

Laboratory experiments, by systematically controlling some of the market parameters, provide simplified environments for understanding performances of information markets. Plott and Sunder[5] set up experiments to examine issues of information aggregation when different traders have diverse information about an underlying state of the world. The information structure did not have aggregate uncertainty, which means that although no individual trader knows the state of nature, but pooling their information together the state can be identified with certainty. Their results demonstrated that market structure was important for information aggregation, and only with appropriate market structure, could markets aggregate diverse information. Lundholm[14] examined the effect of aggregate uncertainty and found that markets aggregated information less efficiently when there was greater aggregate uncertainty. Forsythe and Lundholm[15] found that if participants had heterogeneous preferences, experience of participants was a necessary condition for information aggregation. O'Brien and Srivastava[16] focused on the relationship between asset structure and information aggregation ability of the market. Their results showed that information aggregation ability decreased when asset structure of the market was sufficiently complex. Sunder[17] extensively summarized experimental work on information aggregation. He indicated that the difficulties of the state of research were to understanding what factors facilitate or prevent information aggregation.

Outside the laboratory, many real world online game markets provide test grounds for information markets. The Iowa Electronic Markets (IEM) are real-money online futures markets, in which security payoffs depend on economic and political events such as elections. Presidential election markets of IEM are the most extensively studied. Participants trade securities whose payoffs depend on outcomes of presidential election. Analysis of trading data found that prices in these markets predicted the election outcomes better than polls[18,19,9,10]. Some other online game markets include Hollywood Stock Exchange (HSX)[b], Foresight Exchange (FX)[c], and TradeSports (TS)[d]. HSX trades securities based on events such as future box office proceeds of new movies. FX allows traders to bet on unresolved scientific questions or other claims of public interest. TS offers markets in many domains especially sports. Prices of securities in many real world information markets were found to give as accurate or more accurate predictions than expert opinions[20,21].

Theoretical studies of information markets are relatively rare. Feigenbaum et al.[22] analyzed some computational properties of information markets. They modeled an information market without aggregate uncertainty. The random variable to be predicted is the value of a Boolean function of all individual information. They proved that when the function took a certain form, the information market could accurately predict the value of the function at equilibrium, no matter what the prior distribution of the state of the world was. The number of rounds for the market to converge to the equilibrium equaled the number of traders in the market. Chen et al.[23] studied an information market with aggregate uncertainty. Their analysis showed that a market could converge to equilibrium with any prior distribution of the state of the world and traders information. If the prior distribution satisfied certain conditions, the market was guaranteed to converge to a prediction that aggregated all information of traders. Both of the works were closely related to results of research on common knowledge[24,25,26,27], which provided useful tools to analyze information markets.

3. A Model of Information Market

In order to establish the theoretical underpinnings of information markets, we need to set up realistic models for analysis. There are many possible ways to model information markets, just as many different models for business and financial markets exist. A generic model of information markets should include at least three

[b] http://www.hsx.com/
[c] http://www.ideosphere.com/fx/
[d] http://www.tradesports.com

indispensable components: information structure, market mechanism, and trader behavior.

3.1. *Information Structure*

Information structure of the market specifies what the state space of the world is, how much information traders know about the real state of the world, and how information of traders relates to the real state of the world. Usually information structure of markets is modeled using prior probability distributions of the state of the world and of the information that traders possess.

Let S represents the state space of the world, where $s = (s_1, s_2, ..., s_m) \in S$ is a state vector of m dimensions. Assume there are n traders in the market, where all traders have a common prior probability distribution regarding to state of the world, $\mathcal{P}(s)$: $S \rightarrow [0, 1]$.

The trader's information space is X. Each trader $i = 1, ..., n$ gets a piece of information x_i about the state of the world, where $x = (x_1, x_2, ..., x_n) \in X$ is the information vector for all agents. Traders have common knowledge of the probability distribution of x, conditional on the state of the world s, $\mathcal{Q}(x|s)$: $X \times S \rightarrow [0, 1]$.

Our model attempts to capture aggregate uncertainty, which is common to most real world markets. Market traders are unlikely to possess complete and accurate information about state of the world. Aggregate uncertainty occurs when even if the information from all traders is pooled together, the state of the world is still not fully determined. In our model, aggregate uncertainty stems from the uncertainty of individual information. Markets without aggregate uncertainty, such as that of Feigenbaum et. al[22], can be viewed as special cases of our model.

3.2. *Market Mechanism*

Market mechanism specifies what securities are being traded and trading rules of the market. We model our market as predicting the value of a function $f(s)$. The value of the function is determined by the true state of the world, which will only be revealed some time in the future. One security is traded in the market, whose payoff is contingent on the value of $f(s)$. Specifically, the security pays off $\$f(s)$ in the future. The form of f is common knowledge to all traders.

Following Dubey et al.[28] and Feigenbaum et al.[22], we model the market mechanism as a *Shapley-Shubik market game*[29].The market game proceeds in rounds. Each trader has a_i units of the security at the beginning of each round. In each round, each trader puts up quantities of the security to be sold and simultaneously puts up a positive amount of money to buy the security. For simplicity, we require the traders to offer selling all of their holdings of the security, and assume that there are no restrictions on credit. Then, traders' bids can be represented as a vector $b = (b_1, b_2, ..., b_n)$, where b_i is the amount of money trader i offers to buy securities. Market clears at equilibrium. Thus, the price for a round is $p = \sum_{i=1}^{n} b_i / \sum_{i=1}^{n} a_i$. Only this price p, not individual traders' bids, is publicly announced in each round.

All trading occurs at the market price. At the end of the round, trader i holds the amount $a'_i = b_i/p$ of the security. He or she profits $a_i p$ dollars through selling the security and loses b_i dollars from buying the security. Thus, trader i now has $a_i p - b_i$ dollars. The market then enters a new round, where each agent has the same initial security holdings as previous rounds. The process continues until an equilibrium is reached, after which prices and bids do not change from round to round.

We chose to model information markets as Shapley-Shubik market games rather than using the classical general equilibrium models because both the equilibrium price and the dynamic price formation process are important for understanding information markets. Not only do we want to know that an equilibrium exists, but also that it can be reach. Classical models concentrate on the existence of prices at equilibrium, but do not define prices for off-equilibrium situations, while Shapley-Shubik games have well-defined prices at all time.

In the rest of this paper, we narrow our analysis down to a simple class of information markets, i.e. information markets with Boolean securities. We restrict the function $f(s)$ to be a Boolean function. The corresponding more restricted information structure and market mechanism settings are described as follow. In this class of information markets, $S = \{0,1\}^m$ is the state space of the world. All traders have the same prior probability distribution of the true state of the world, $\mathcal{P}(s)$: $\{0,1\}^m \to [0,1]$. The market wants to predict the value of the function $f(s) : \{0,1\}^m \to \{0,1\}$, the form of which is common knowledge to traders. The information space is $X = \{0,1\}^n$. Each trader i, $i = 1, ..., n$, gets a piece of information x_i, which is either 0 or 1, about the true state of the world. The information that traders get does not fully reveal the true state of the world. It relates to the true state of the world according to the probability distribution $\mathcal{Q}(x|s)$: $\{0,1\}^n \times \{0,1\}^m \to \{0,1\}$, which is also common knowledge across traders. Market participants trade a security that pays off \$1 if $f(s) = 1$ and \$0 if $f(s) = 0$.

3.3. *Trader Behavior*

Modeling trader behavior is to model trader's risk preference and rationality, which directly lead to trader's trading strategies. We assume that market traders are risk neutral and myopic. Their utility in each round is the sum of the expected payoff of their security holdings and their money holdings. Thus, for trader i who submits bid b_i in a round, his or her utility is:

$$U_i(b) = Pr_i(f(s) = 1) \times (b_i/p + a_i p - b_i) + Pr_i(f(s) = 0) \times (a_i p - b_i), \qquad (3.1)$$

where $Pr_i(f(s) = 1)$ and $Pr_i(f(s) = 0)$ are trader i's subjective probability assessments about the value of $f(s)$. They usually are assessments conditional on trader i's information. A trader's utility in each round is a function of bids of all traders, because the market clearing price is determined by all submitted bids. Traders are myopic, thus they only care their utility of the current trading round.

Although real world economic agents are risk averse, it is still reasonable to model traders in information markets as risk neutral. This is because given the utility function of a risk averse trader and his or her subjective probability distribution about some random variable, we can always transfer them to a risk neutral utility function and risk neutral probability distribution [30]. A hypothetical risk neutral trader with the transformed utility function and probability distribution will behave exactly the same as the original risk averse trader. Thus, without lose of generality, we can model traders of information markets as risk neutral.

We do not put any restriction on trader rationality in the model, because we want our model to capture the most fundamental part of information markets. Different assumptions of trader rationality result in different trading strategies. We construct our analysis in the next section according to different assumptions of trader strategy.

4. Conditions for Making the Most Desirable Predictions

In this section, we examine information markets with Boolean securities for their ability to reach consensus among traders and to fully aggregate information across all traders. For simplicity, we further assume that each trader has one unit of the security, i.e. $a_i = 1$, at the beginning of each trading round and is forced to put it for sale in the market. We are interested in investigating under what conditions our information market model converges to the equilibrium where information of traders is fully aggregated. The examination is performed respectively under three different assumptions of trader behavior.

4.1. *Direct Communications Equilibrium*

We discuss the concept of direct communications equilibrium in this section as a benchmark for further evaluating performance of information markets. In stead of making the price public in the market, market traders can directly reveal their private information to each other. In this situation, all private information becomes public to all traders, hence all traders have the same expectation about the value of the security and an equilibrium can be reached immediately. This kind of equilibrium is called a direct communications equilibrium [25]. The equilibrium market price equals the expectation of the security payoff conditional on all available information, i.e. $E(f(s)|x)$. Since at direct communications equilibrium, the market price incorporates all available information, it is the best prediction that an information market can possibly converge to.

4.2. *Information Markets with "Truth-Telling" Traders*

One of the simplest ways to model trader bidding behavior is to assume that traders bid truthfully, that is, each trader in each round bids his or her current expected payoff of a unit of the security. Expectations are calculated based on probability distribution of the state of the world $\mathcal{P}(s)$, conditional probability distribution of information $\mathcal{Q}(x|s)$, and the information obtained from previously announced market prices. As available information changes when market proceeds, traders revise their expectations accordingly.

Assuming that traders are "truth-telling" seems reasonable when the number of traders in the market is relatively large and complicated strategic reasoning might not effectively improve a trader's utility over simply bidding one's true valuation.

With "truth-telling" traders, our model is the same as that in Chen et al.[23]. We briefly restate the main result on convergence properties of information markets using our notation as the following theorem.

Theorem 1. *(Chen et al.[23]) If (a) x_i's are independently drawn, and their probability distributions conditional on s, i.e. the marginal probability distributions of $\mathcal{Q}(x|s)$, denoted as $q(x_i|s)$'s, are identical for all $i = 1, ..., n$, and (b) traders' expectations of the security value are different with different private information, i.e. $Pr(f(s) = 1|x_i = 0) \neq Pr(f(s) = 1|x_i = 1)$, an information with "truth-telling" traders converges to direct communications equilibrium within two rounds of trading. At equilibrium, each trader's expectation of the security payoff equals the security price.*

Theorem 1 demonstrates that if the conditional distribution of traders' private information, $\mathcal{Q}(x|s)$, satisfies certain conditions, an information market is guaranteed to converge to the equilibrium that aggregates all information in the market.

4.3. *Information Markets with Bayesian Traders*

When the information market is relatively small, having only a few traders, it is no longer reasonable to assume that traders will truthfully bid their expectations of the security payoff in each round of trading. Thus, we make the assumption that traders are fully rational and bid to maximize their expected utilities in each round. We use the term "Bayesian traders" to represent these expected utility maximizers.

Under this assumption, each trading round of the information market can be viewed as a Bayesian game. We can show that if the game has a Bayesian Nash Equilibrium, Theorem 1 is also valid for information markets with Bayesian traders. We state the results in Corollary 1.1, and provide a sketch of the proof in this section. Details of the proof for Corollary 1.1 is shown in Appendix A.

Corollary 1.1. *If the following three conditions are met, an information market with Bayesian traders converges to direct communications equilibrium within two rounds of trading. At equilibrium, each trader's expectation of the security payoff equals the security price.*

(a) *x_i's are independently drawn, and their probability distributions conditional on s, i.e. the marginal probability distributions of $\mathcal{Q}(x|s)$, denoted as $q(x_i|s)$'s, are identical for all $i = 1, ..., n$.*
(b) *Traders' expectations of security value are different with different private information, i.e. $Pr(f(s) = 1|x_i = 0) \neq Pr(f(s) = 1|x_i = 1)$.*

(c) The Bayesian Nash Equilibrium exists for the Bayesian game in each round.

Sketch of Proof: Because traders only have incomplete information - they do not know others' private information, a Bayesian trader's expected utility before a trading round is a function of the trader's own information, trader's belief about other traders information, the trader's own bidding strategy, and other traders' bidding strategies. Let $(b_i(0), b_i(1))$ represents a biding strategy for trader i, where $b_i(0)$ represents trader i's bidding strategy when the private information x_i is 0 and $b_i(1)$ represents trader i's bidding strategy when the private information x_i is 1. Thus, the optimal response functions for trader i are obtained by setting the partial first order derivatives of the expected utility with respect to $b_i(0)$ to 0 when x_i is 0, and with respect to $b_i(1)$ to 0 when x_i is 1. The best responses of trader i are functions of bids of all traders. Because $q(x_i|s)$'s are independent and identical for all $i = 1, ..., n$, traders are symmetric, which means that the optimal strategy for all traders should be the same at the Bayesian Nash Equilibrium. This reduces the best response functions to two equations with two variables $b_i^*(0)$ and $b_i^*(1)$. It can then be proved that $b_i^*(0) = b_i^*(1)$ only if $Pr(f(s) = 1|x_i = 0) = Pr(f(s) = 1|x_i = 1)$. In other words, condition (b) in Corollary 1.1 guarantees that $b_i^*(0) \neq b_i^*(1)$. Thus, traders can infer what information the others have from the price of the first round. The information market converges to the direct communications equilibrium in the second round.

The existence of Bayesian Nash equilibrium depends on the prior probability distributions $\mathcal{P}(s)$ and $\mathcal{Q}(x|s)$. Even if it exists, finding the equilibrium strategy can be computationally complex.

4.4. *Information Markets with Bounded Rational Bayesian Traders*

Fully rational Bayesian traders need to consider other traders' infinite hierarchies of beliefs and form consistent beliefs over them. It is unlikely that traders would be smart enough to consider such a space of infinite beliefs. Thus, we assume *bounded rationality*[31] of traders in this part of analysis.

There are many different ways to model bounded rationality[32]. Without discussing which one is the most appropriate, which is still an open question, we model our bounded rational Bayesian traders as: Each trader forms beliefs about other traders' information and attempts to maximize their expected payoff, but he or she at the same time believes that other traders bid truthfully. This conforms to the understanding of bounded rationality in Bernheim's paper[33]: "We might not expect agents to check the consistency of their beliefs for more than a finite number of levels." In our case, market traders only check the consistency of their beliefs for one level. Information markets with this kind of traders still converge to the direct communications equilibrium when certain conditions are satisfied. We state the result as Corollary 1.2.

Corollary 1.2. *If the following three conditions are met, an information market with bounded rational Bayesian traders converges to direct communications equilibrium within two rounds of trading. At equilibrium, each trader's expectation of the security payoff equals the security price.*

(a) x_i*'s are independently drawn, and their probability distributions conditional on* s*, i.e. the marginal probability distributions of* $\mathcal{Q}(x|s)$*, denoted as* $q(x_i|s)$*'s, are identical for all* $i = 1, ..., n$*.*
(b) Traders' expectations of security value are different with different private information, i.e. $Pr(f(s) = 1|x_i = 0) \neq Pr(f(s) = 1|x_i = 1)$*.*
(c) It is optimal for a trader to bid some positive value.

The proof for Corollary 1.2 is similar to that for Corollary 1.1, but traders face less complicated optimization problems in choosing their bids. Condition (c) in Corollary 1.2 is to ensure that traders have a positive optimal strategy. This depends on the prior probability distributions $\mathcal{P}(s)$ and $\mathcal{Q}(x|s)$ again, but it is less strict than the condition that requires the Bayesian Nash equilibrium exist as in Corollary 1.1. The results for three different types of traders are summarized in Table 1.

5. Discussions and Implications

In section 4, we proved some fundamental properties on when information markets can converge to the direct communications equilibrium, which aggregates all information across traders and is the best possible prediction for information markets. Specifically,

(1) With "truth-telling" traders, sufficient conditions for an information market to converge to the direct communications equilibrium are that (1) distributions of individual trader's information conditional on the state of the world are identical and independent, and (2) traders' expectations of the security value are different with different private information;
(2) With fully rational Bayesian traders, in addition to the two conditions listed in item 1, we need that the Bayesian Nash equilibrium exists in each round, to guarantee that the information market converges to the direct communications equilibrium;
(3) With bounded rational Bayesian traders, the existence of positive optimal bid for traders is needed, in addition to the two conditions listed in item 1, to guarantee that the information market converges to the direct communications equilibrium.

The existence of Bayesian Nash equilibrium and the existence of positive optimal bids for traders all depend on the prior distributions of the state of the world and the distribution of traders' information conditional on the state of the world. An implication of our results is that, in order for information markets to aggregate

Table 1. Convergence Conditions and Trader Behavior

Comparison Items	Trader Types		
	Truth-Telling	Bayesian	Bounded Rational
Trader Behavior	Traders truthfully bid their conditional expectations of the security value.	Traders are fully rational and bid to maximize their expected utility in each round.	Each trader attempts to maximize their expected payoff, but at the same time believes that other traders truthfully bid their conditional expectations of the security value.
Convergence Conditions	The marginal probability distributions of $\mathcal{Q}(x\|s)$, $q(x_i\|s)$'s, are independent and identical for all $i = 1, ..., n$.	Same.	Same.
	Traders' expectations of security value are different with different private information.	Same.	Same.
	None.	The Bayesian Nash Equilibrium exists for the Bayesian game in each round.	It is optimal for a trader to bid some positive value.

all information and converge to the direct communications equilibrium, they need to be properly designed. Special care is needed to achieve the desired information structure of the market.

Laboratory experiments using human subjects can be used to examine the validity of our model in settings that are more realistic but still maintain close parallel to the theory. Results of our initial experiments on information markets without aggregate uncertainty, which will be discussed in a separate paper, support theory to some extent. We expect to examine markets with aggregate uncertainty experimentally to get deeper understanding of the model.

Over the past years, many real world online game markets provide test grounds for information markets. Several companies have also deployed information markets to assist business decision making[34,35]. For example, Siemens Austria set up two information markets to predict whether a project can be finished at the date planned and whether it will be finished earlier or later than planned[35]. People from the software department were motivated to participate in the markets. Market results showed satisfying potential to forecast the project finish time and hence supporting decisions in project management. Inside Hewlett-Packard Corporation, an information market for the purpose of making sales forecast was running[34]. Information markets, if properly designed, have substantial potential to help organizations make better informed decisions. Major issues considered in information market design include the choice of security structure, design of trading mechanism, selection of incentive/reward method, user interface design, and methods of legal and privacy resolution[6,7,36].

6. Conclusion

With the fast growth of the Internet, information markets have recently emerged as a promising alternative forecasting tool. Despite merits and popularity of information markets, why they work, how they work, when they work, and how to design effective information markets are still open questions to a large extent.

This paper is an initial step toward our long-term goal of understanding the use of information markets as tools to predict uncertain decision variables. We have introduced information markets as a promising mechanism for predicting uncertain variables that are related to decision making. A generic model of information markets with aggregate uncertainty has been established. By examining a simple class of information markets based on our model with three different assumptions of trader strategies, we have proved that some fundamental properties on when information markets can converge to the direct communications equilibrium, which aggregates all information across traders and is the best possible prediction for information markets.

However, in order for information markets to aggregate all information and converge to the direct communications equilibrium, they need to be properly designed. In the future, we are interested in investigating the following issues: (1) design issues of information markets. Our results have shown that information markets might not always converge to the direct communications equilibrium, where all information is aggregated. It is important to study how to design information markets to ensure good predictions; (2) modeling of bounded rationality of traders. In this paper, we model the bounded rationality of traders in a very simple way. We will explore more realistic ways to model trader's rationality and analyze the properties of information markets accordingly; and (3) real world applications. We are interested in applying information market technology to the real world, especially in financial market prediction, supply chain management, and technological forecasting.

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Appendix A. Proof of Corollary 1.1

Consider the first round of the market, let $b = (b_1, b_2, ..., b_n)$ be the bid vector of market traders. We use b_{-i} to represent all bids of traders other than trader i. The market price is $p = \sum_{i=1}^{n} b_i/n$. The net trade of the security for trader i is $(b_i/p)-1$, while net money income for trader i is $p - b_i$. Thus, the utility of trader i is:

$$\begin{aligned} U_i(b) &= Pr(f(s) = 1|x_i) \times (b_i/p - 1) + p - b_i \\ &= Pr(f(s) = 1|x_i) \times (\frac{nb_i}{\sum_{j=1}^{n} b_j} - 1) + \frac{\sum_{j=1}^{n} b_j}{n} - b_i, \end{aligned} \tag{A.1}$$

which is a function of the biding vector b. It not only depends on trader i's own bid but also depends on all other traders' bids.

Let $(b_i(0), b_i(1))$ represents a biding strategy for trader i, where $b_i(0)$ represents the trader i's bidding strategy when the private information x_i is 0 and $b_i(1)$ represents the trader i's bidding strategy when the private information x_i is 1. Both $b_i(0)$ and $b_i(1)$ should be positive real values. Without loss of generality, we consider the cases for trader 1. When $x_1 = 0$, trader 1's expected utility is

$$\begin{aligned} E_1(U_1(b_1(0), b_{-1})) = &Pr(\textstyle\sum_{j=1}^{n} x_j = 0|x_1 = 0)U_1(b_1(0), b_2(0), ..., b_n(0)) \\ &+Pr(\textstyle\sum_{j=1}^{n} x_j = 1|x_1 = 0)U_1(b_1(0), b_2(1), ..., b_n(0)) \\ &+\ldots\ldots \\ &+Pr(\textstyle\sum_{j=1}^{n} x_j = n-1|x_1 = 0)U_i(b_1(0), b_2(1), ..., b_n(1)). \end{aligned} \tag{A.2}$$

In equation (A.2), $Pr(\sum_{j=1}^{n} x_j = i|x_1 = 0)$ represents agent 1's probability assessment about the event that there are i traders in the market whose information is 1. Without loss of generality, we further assume that, they are trader 2 through trader $i+1$. The utility terms are of the form as in equation (A.1). Similarly, if $x_1 = 1$, trader 1's expected utility is

$$\begin{aligned} E_1(U_1(b_1(1), b_{-1})) = &Pr(\textstyle\sum_{j=1}^{n} x_j = 1|x_1 = 1)U_1(b_1(1), b_2(0), ..., b_n(0)) \\ &+Pr(\textstyle\sum_{j=1}^{n} x_j = 2|x_1 = 1)U_1(b_1(1), b_2(1), ..., b_n(0)) \\ &+\ldots\ldots \\ &+Pr(\textstyle\sum_{j=1}^{n} x_j = n|x_1 = 1)U_i(b_1(1), b_2(1), ..., b_n(1)). \end{aligned} \tag{A.3}$$

Trader 1 maximizes his or her expected utility under the constrain that $b_1(0)$ and $b_1(1)$ are positive. If the Bayesian Nash equilibrium exists, then at equilibrium the following first order conditions must be satisfied.

$$\begin{cases} \partial E_1(U_1(b_1^*(0), b_{-1}^*))/\partial b_1^*(0) = 0 \\ \partial E_1(U_1(b_1^*(1), b_{-1}^*))/\partial b_1^*(1) = 0 \end{cases} \tag{A.4}$$

Because market traders have the same probability distributions, $q(x_i|s)$'s, their behavior should be symmetric. Hence, their optimal strategies must be the same at equilibrium, i.e., $b_1^*(0) = b_2^*(0) = ... = b_n^*(0)$ and $b_1^*(1) = b_2^*(1) = ... = b_n^*(1)$. Then (A.4) becomes:

$$\begin{cases} Pr(f(s) = 1|x_1 = 0) \sum_{i=0}^{n-1} Pr(\sum_{j=1}^{n} x_j = i|x_1 = 0)G_i + \frac{1}{n} - 1 = 0 \\ Pr(f(s) = 1|x_1 = 1) \sum_{i=1}^{n} Pr(\sum_{j=1}^{n} x_j = i|x_1 = 1)H_i + \frac{1}{n} - 1 = 0 \end{cases} \tag{A.5}$$

where

$$G_i = \frac{n}{(n-i)b_1^*(0) + ib_1^*(1)} - \frac{nb_1^*(0)}{((n-i)b_1^*(0) + ib_1^*(1))^2}, \tag{A.6}$$

$$H_i = \frac{n}{(n-i)b_1^*(0) + ib_1^*(1)} - \frac{nb_1^*(1)}{((n-i)b_1^*(0) + ib_1^*(1))^2}, \qquad \text{(A.7)}$$

Optimal bidding strategy for trader 1 (also for other traders) is the solution to equation system (A.5).

Because $Pr(f(s) = 1|x_i = 0) \neq Pr(f(s) = 1|x_i = 1)$, $b_1^*(0)$ does not equal to $b_1^*(1)$. Hence, after the first round of the trading, each trader can infer, from the market price, how many other traders bid $b_1^*(0)$ and how many bid $b_1^*(1)$. The total number of traders that have information 1 and total number of traders that have information 0 then become known to all market traders. Every trader has the same prediction about the value of the security now. The game of the second round of the market reduces to a game with complete information, Nash equilibrium of which indicates that every trader would bid his true valuation. The information market converges to direct communications equilibrium in the second round.

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